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TWO-DIMENSIONAL VAPOR FLOW ANALYSIS IN HEAT PIPES

by

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Introduction

The analysis of the vapor flow in heat pipes is a complex problem. It has been solved only for special cases requiring assumptions such as incompressible flow, constant channel cross-section, large length-to-diameter ratio, very small radial Reynolds numbers or very large ones, laminar flow, and constant heating and cooling rates. Usually, flow conditions in real heat pipes do not correspond to such idealized cases. For this reason the computer code "AGATHE" was developed.

The code is intended to evaluate axially symmetric heat pipes with compressible vapor flow at Mach numbers up to 1 and at all radial Reynolds numbers. The code can be used to evaluate empirical factors describing turbulence. Furthermore, heat input and output is modeled by describing liquid heat transfer loops. This method leads to nonuniform heating and cooling rates typical of actual heat pipes. Presently the code is adapted to evaluate heat pipes in tubular geometry composed of a series of heat transfer and adiabatic zones of cylindrical or conical shape.

In this analysis the two-dimensional mathematical problem was reduced to a number of ordinary differential equations, which are integrated by a Runge-Kutta scheme. The reduction was achieved, first, by starting from the Navier-Stokes equation using the boundary layer approximation; this approximation introduces the main limitation of the code, restricting its use to the calculation of vapor ducts with large length-to-diameter ratios. Second, the velocity profile was simulated by a power series. The n coefficients of this series were determined such that at each axial position the radial pressure gradient was approximately zero, as specified by the boundary layer approximation.

For testing the code, calculations for cylindrical heat pipes were done up to the sonic limit. These results have been reported.[1] In the limiting case of large radial Reynolds numbers good agreement with analytical predictions was obtained. It was found that the flow rates of the heat transfer loops can strongly influence the radial heat flux distribution.

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For example, the radial heat transfer can be fairly uniform or concentrated on a fraction of the condenser surface. Such an effective condenser shortening can lead to considerable deterioration of the effective radial heat transfer coefficient. This effect is very important for nonlinear heat rejection such as found in radiation coupled heat pipes. This code is a very useful instrument for both analytical prediction and evaluation of experimental data.

Modeling of the Heat Pipe

Uncertainties exist in describing the vapor flow in heat pipes where inertia effects dominate the flow. This occurs at Radial Reynolds numbers greater than one where

$$Re_r = - \frac{D\rho v}{2\mu} \tag{1}$$

In addition high Mach number, compressible flows which occur during the start-up of heat pipes from low vapor pressures have not been investigated.

The difficulties in analyzing the vapor flow in heat pipes at $Re_r > 1$ and at Mach numbers close to 1 are, in part, of mathematical origin. There is a pronounced nonsimilarity of the velocity profiles at different cross sections. At low $\bar{M}$, the nonsimilarity is essentially limited to the condenser, at high $\bar{M}$ it also occurs in the evaporator. In any case, a two-dimensional, compressible flow analysis is required. A further complication arises at large Re_r due to the large slope and curvature of the velocity profiles in the condenser and also in the evaporator at high $\bar{M}$. Only a few studies have been published in this field.[2, 3, 4]

The other difficulties are more of a physical nature. They arise from the turbulence which occurs in cylindrical condensers at high Re_r even if the axial Reynolds number is well below the transition value for pipe flow. According to the criteria of Raleigh and Tollmien such flows are unstable.

Inflection points in heat pipe velocity profiles occur in cylindrical condenser sections. The inflection point first appears at $Re_r = -1$ near the end of the condenser. At larger Re_r inflection points occur in all the downstream profiles in the condenser, and the beginning of this region moves upstream with increasing Re_r. A detailed experimental and theoretical study[5] on a cylindrical, porous tube showed that turbulence exists in the downstream part of the tube. The existence of turbulence forces the condenser analysis to an empirical basis.

The analysis of the vapor flow in heat pipes operating at high Mach numbers and $Re_r > 1$ led to the development of the computer code AGATHE. The code is intended to provide a versatile tool to aid in the design and analysis of the vapor-limited heat pipes.

The Computer Code

The code is intended to evaluate heat pipes with compressible vapor flow at Mach numbers close to one and at all radial Reynolds numbers. Presently the code is adapted for heat pipes in tubular geometry composed of a series of heat transfer and adiabatic zones of cylindrical or conical shape. It is also usable for calculating the channel shape of adiabatic zones operating at constant pressure gradient, which is of interest for designing nozzles and diffusers.

For each zone a laminar two-dimensional calculation or a one-dimensional calculation with input of empirical factors for describing turbulence can be chosen. The most convenient choice of these factors can be resolved only when more data on turbulent heat pipe flows become available.

The code contains a model of the heat input and output using external liquid loops. This makes it possible to analyze nonuniform heat addition and removal, an important consideration in radiation coupled heat pipes.

Vapor Model

The vapor is described as an isothermal perfect gas with constant heat of vaporization or

$$\frac{p}{\rho} = c^2 = \text{const.} \tag{2}$$

$$h_{fg} = \text{const.} \tag{3}$$

This model is strictly valid for analyzing heat pipes which use organic working fluids of high molecular weight. Such vapors have a very high molar heat capacity, which keeps them nearly isothermal during expansion or compression. As a result such vapors superheat on expansion (rather than supersaturate like vapors with molecules composed of only few atoms). However, the isothermal perfect gas assumption is a reasonable approximation even for these latter cases. The temperature of the gas is approximated by the average temperature of the liquid-vapor interface in the evaporator.

Flow Model

The vapor flow is described by the conservation of mass

$$\frac{\partial}{\partial r}(r\rho u) + r\frac{\partial}{\partial z}(\rho w) = 0 \tag{4}$$

and the conservation of momentum using Prandtl's boundary layer approximation

$$\frac{\partial P}{\partial z} = -\rho\left(u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z}\right) + \frac{\mu}{r}\frac{\partial}{\partial r}\left(r \frac{\partial w}{\partial r}\right) \qquad (5)$$

$$\frac{\partial P}{\partial r} = 0 \qquad (6)$$

The boundary layer approximation allows a considerable reduction in the computation effort for solving the flow problem.

Solution Procedure

The velocity profile is approximated by a power series

$$w = \bar{w}(z) \sum_{i=1}^{n} a_i(z)\eta^{i-1} \qquad (7)$$

with

$$\eta = \left(\frac{2r}{D}\right)^2 \qquad (8)$$

The n velocity profile coefficients, a_i must be determined. Two are given by the conditions

$$\sum_{i=1}^{n} a_i = 0 \qquad (9)$$

and

$$\sum_{i=1}^{n} \frac{a_i}{i} = 1 \qquad (10)$$

Condition (9) follows from (7) together with the no-slip condition at the wall and condition (10) is obtained by averaging (7) over the cross section.

Eliminating u and inserting (7) into (5) gives for $\partial P/\partial z$ a power series in η of order 2(n-1). But according to (6) $\partial P/\partial z$ must be independent of η. So all the 2(n-1) factors of the η-powers must be zero, which gives 2(n-1) differential equations for $\partial P/\partial z$ and the

a_i's. But from (7) there are only n-2 coefficients, a_i. Therefore with (7) only an approximate solution of the flow equations can be obtained. The a_i's are chosen so that the radial variation of $\partial P/\partial z$ is minimized.

One possibility is to fulfill only the first n-1 of these differential equations, which makes $\partial P/\partial z$ very constant in the center part of the flow channel but leads to relatively large errors at the wall. One can improve the quality of the solution at the wall by substituting for the last one of these differential equations an equation for $\partial P/\partial z$ at the wall. The resulting set of differential equations is integrated using a Runge-Kutta scheme.

The power series approximation for the velocity profile has proven to be sufficiently flexible to analyze a broad range of heat pipe operating conditions. AGATHE was used to calculate the vapor pressure drop in cylindrical evaporators which is presented below.

For an analysis of the pressure drop it is convenient to separate the contributions of the inertia forces and the viscous forces. Averaging the momentum equation (5) over the cross section leads to

$$\frac{dP}{dz} = - \frac{d}{dz} (\beta \rho \bar{w}^2) - \frac{2 f_F \rho \bar{w}^2}{D} \qquad (11)$$

where the momentum factor β is defined as

$$\beta = \int_0^1 \left(\frac{w}{\bar{w}}\right)^2 d\eta \qquad (12)$$

and f_F is the Fanning friction factor

$$f_F = - \frac{8}{Re} \left(\frac{\partial}{\partial \eta} \left(\frac{w}{\bar{w}}\right)\right)_{\eta=1} \qquad (13)$$

with the axial Reynolds number

$$Re = \frac{\rho \bar{w} D}{\mu} \qquad (14)$$

Integrating (22) along the evaporator gives

$$\left(1 - \frac{P}{P_o}\right)\frac{P}{P_o} = \left(\frac{q}{q_{so}}\right)^2 \left[\beta + \frac{\varphi}{8Re_r}\right] \qquad (15)$$

where φ is defined by

$$\varphi = \frac{2}{\bar{M}L_e}\int_0^{L_e} f_F Re\bar{M}dz \qquad (16)$$

The two terms in brackets in (15) are the inertia contribution and the wall-friction contribution to the axial pressure drop, respectively, while β and φ are universal functions of q/q_{so} and Re_r which are given in Figs. 1 and 2.

At low heat fluxes β varies between 1.234 and 1.333 which are the values for the cosine profile (at $Re_r \to \infty$).

$$\frac{w}{\bar{w}} = \frac{\pi}{2}\cos\left(\frac{\pi}{2}\eta\right) \qquad (17)$$

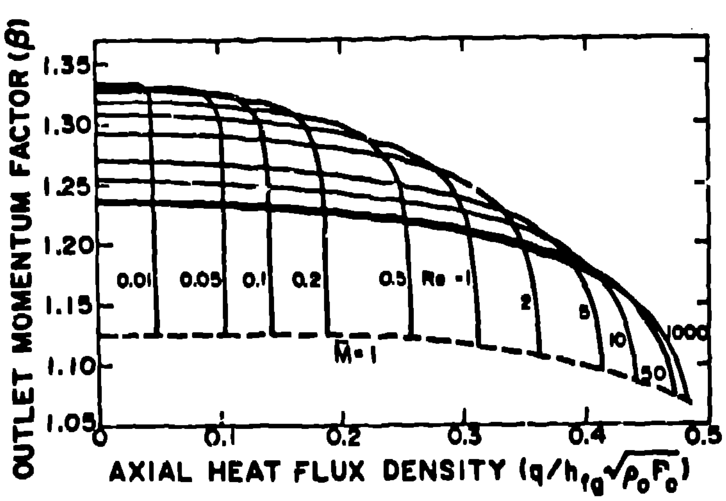

Fig. 1. Outlet momentum factor for cylindrical heat pipe evaporators

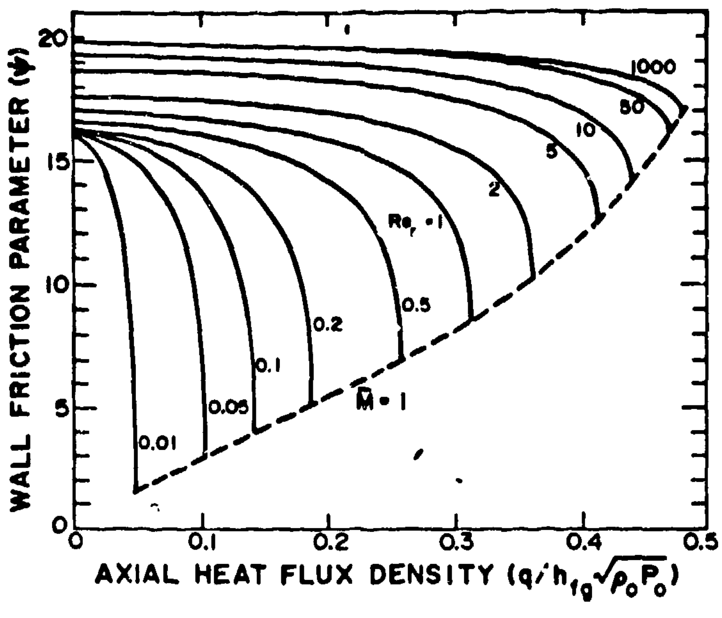

Fig. 2. Wall friction parameter for cylindrical heat pipe evaporators

and the Poiseuille profile (at $Re_r \to 0$)

$$\frac{w}{\bar{w}} = 2(1-\eta) \tag{18}$$

respectively. With increasing heat flux the values of β decreases in correspondence with the flattening of the velocity profile. At low Re_r the transition to the choked profile occurs very abruptly, after the profile has remained nearly constant throughout most of the evaporator.

The parameter for the wall friction, φ, shown in Fig. 2 has behavior similar to β. At low heat fluxes φ becomes identical with $f_F Re$ of the inlet profile, the values of which range from 16.0 for the Poiseuille profile to 19.74 for the cosine profile. With increasing heat flux φ decreases but, contrary to the β behaviour, the variation at small Re_r is relatively large.

The data presented are based on the boundary layer approximation which is only valid for large $Le/_D$ ratios. Therefore the diagrams should not be used if Le is less than one diameter.

Conclusions

The code AGATHE is a versatile instrument for both analytical prediction and evaluation of experimental data. Its relative simplicity is largely due to the utilization of the boundary layer approximation. Based on comparison with other two dimensional analyses and test data, the use of the boundary layer approximation in analysing compressible vapor flows in cylindrical heat pipes appears to be justified.

Acknowledgement

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Nomenclature

a_i	velocity profile coefficient
A	cross sectional area
c	velocity of sound in the vapor
D	diameter of the vapor channel
f_F	Fanning friction factor
h_{fg}	specific heat of vaporization
L	length of the heat pipe
L_e	length of the evaporator
$\bar{M}$	Mach number, averaged over the cross section
n	number of velocity profile coefficients
P	vapor pressure
q	axial heat flux density
q_{so}	axial heat flux density at sonic vapor speed
Q	axial heat flow
r	radial coordinate
Re	axial Reynolds number
Re_r	radial Reynolds number
T_i	temperature at the liquid-vapor interface
u	radial component of the vapor velocity
v	velocity component in direction of the outer normal at the wall of the vapor channel
w	axial component of the vapor velocity
$\bar{w}$	axial velocity component, averaged over the cross section
z	axial coordinate
β	momentum factor
η	dimensionless radial coordinate
μ	viscosity of the vapor
ρ	vapor density
ρ_f	density of the coolant
φ	parameter for the wall friction

Subscripts

o	evaporator inlet